Adaptive Control of a Chemical Process System

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Modification of parameters in the conventional controller equation usually is necessitated by uncontrollable process parameter drifts which occur during the operation of the large scale continuous processes typically found in the chemical industry. To this end, a normalized version of the model reference adaptive control system, including suitable procedures for adjusting the adaptive loop gains, was developed and demonstrated to provide excellent adaptive performance for a single concentration control loop for a simulated stirred-tank chemical reactor. The three constants in a conventional PID controller were simultaneously adjusted to accomplish this adaptation. Equally excellent adaptive performance was achieved in an interacting control loop scheme simulated by control of both the concentration and the temperature in the same stirred-tank reactor. In both cases adaptation was achieved in a time interval which was sufficiently short to make this method of adaptive control feasible for use in a real processing situation. Models needed for implementation of the model reference adaptive control scheme were easily developed. In a real application these models could be easily developed from studies which could take place during prestartup or early operations of the process; hence no a priori knowledge would be required. A wide range of adaptive loop gains was demonstrated to provide a stable overall control system. A high degree of stability was demonstrated for the overall system despite the initiation of large extraneous load upsets occurring during the operation of the adaptive control system.

The method presented has not been rigorously proven to be applicable to every real control problem requiring adaptive control. However, the method was demonstrated to be highly effective over a wide range of required controller adjustments in cases similar to those encountered in large continuous processes. Thus the method merits serious consideration for possible implementation whenever adaptive control is required.

The availability of the conventional PID (proportional integral derivative) controller has permitted the chemical engineer to design control systems satisfactorily for most complex plants despite incomplete knowledge. During prestartup or early operation of such a plant, the parameters available for adjustment in the PID controllers are set, often by trial and error, to provide the plant with good dynamic performance in the presence of the inevitable process upsets. However, uncontrollable parameters which can vary slowly with time, such as catalyst activity, may exist. These parameters could vary in such a way that as the process is operated, the overall dynamic performances of the control systems might be adversely affected if some technique to retune the controller parameters is not employed. Adaptive control addresses itself to this particular problem. An adaptive control system can be defined as an on-line systematic technique for modifying the values of available adjustable parameters, such as those on a conventional controller, to maintain good overall control system performance in spite of uncontrolled and unmeasurable process parameter changes.

An adaptive control system is a system which has been provided with a means for continuously monitoring its own performance in relation to a given figure of merit and a means of modifying its own parameters so as to approach this criterion (*1*). The methods of adaptive control may be classified into three broad categories: (1) High gain in the forward path of the control loop (*2*, *3*). (2) Parameter adjustment preprogrammed against measured operating conditions (*1*, *4* to *8*). (3) Parameter adjustments based on certain performance indices in the absence of detailed knowledge about the parameters and mathematical relationships of the overall system (*9* to *13*). For those interested in more background concerning these techniques than is provided by the references listed, a more complete bibliography is given by Braun and Truxal (*14*).

High gain techniques do not lend themselves readily to the situation generally found in the chemical industry because they require operation on the verge of instability. Methods in the second category are of limited applicability because preprogramming of the parameter adjustments based on some identified process behavior function requires a rather comprehensive knowledge of the equations of the system. These would not generally be available. Techniques which fall in the third category are applicable to the adaptive control problem of the chemical industry. This paper discusses the application of a model reference adaptive control system to the problem of adjusting the parameters in conventional PID controllers to compensate for process parameter changes in a simulated chemical industry processing situation (*15*). Results for both a single-control loop and an interacting control situation are discussed.

SIMULATION OF THE CHEMICAL ENGINEERING ADAPTIVE CONTROL PROBLEM

The stirred-tank reactor system of Kermode and Stevens was used to represent a typical chemical processing system (*16*). This system is shown in Figure 1. A list of appropriate values taken from Kermode and Stevens' paper is

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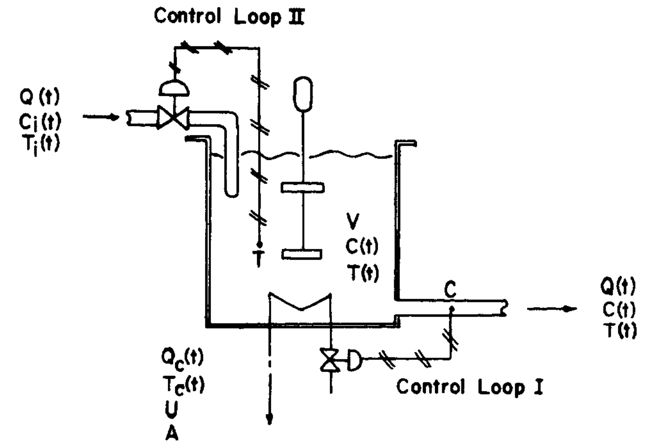

Fig. 1. Chemical reactor system.

presented in Table 1. A single exothermic reaction took place.

$$A \xrightarrow{k(T)} B \tag{1}$$

Assuming that the volume of the reactor remained constant, a mass balance on reactant yielded

$$\frac{dC}{dt} = \frac{Q}{V} C_i - \frac{Q}{V} C - ke^{-E/RT} C \tag{2}$$

Assuming that the accumulation of energy in the cooling coil was negligible and that the arithmetic mean temperature could be used to determine heat transfer from the reactor to the coil, the energy balance was

$$\frac{dT}{dt} = \frac{Q}{V} T_i - \frac{Q}{V} T + \frac{(-\Delta H) e^{-E/RT}}{\rho C_p} \times C - \frac{UA}{\rho C_p V} \frac{2\rho_c C_{p_c} Q_c}{2\rho C_{p_c} Q_c + UA} (T - T_c) \tag{3}$$

The above process was to be controlled at the unstable operating point found by Kermode and Stevens. Two control schemes were studied. The first was the single loop in which the outlet concentration of reactant was controlled by the flow of cooling water. This is shown as control loop I in Figure 1. In the single-loop situation, the flow rate of reactant was held constant. The second was an interacting control scheme resulting from simultaneous control of the concentration and the temperature in the reactor. Temperature was controlled by the flow of reactant. Thus control loops I and II were in operation simultaneously.

TABLE 1. STEADY STATE VALUES OF PROCESS PARAMETERS

A	500 sq. ft.	
C	0.276 lb.-moles/cu. ft.	
C_i	0.500 lb.-moles/cu. ft.	
C_p	1.0 B.t.u./(lb.)(°R.)	
C_{p_c}	1.0 B.t.u./(lb.)(°R.)	
E	45,000 B.t.u./lb.-mole	
ΔH	−20,000 B.t.u./(lb.)(°R.)	
k_o	3.0×10^{11} sec.$^{-1}$	
Q	0.500 cu. ft./sec.	(single-control loop)
	0.329 cu. ft./sec.	(interacting control loop when $ACT = 1.0$)
Q_c	0.200 cu. ft./sec.	(single-control loop when $ACT = 1.0$)
	0.121 cu. ft./sec.	(interacting control loop when $ACT = 1.0$)
R	1.987 B.t.u./(lb.-mole)(°R.)	
T	709.18°R.	(single-control loop when $ACT = 1.0$)
	700.0°R.	(interacting control loop)
T_i	690.0°R.	
T_c	520.0°R.	(before drift)
U	100 B.t.u./(hr.)(sq. ft.)(°R.)	
V	100 cu. ft.	
ρ	60.0 lb./cu. ft.	
ρ_c	60.0 lb./cu. ft.	

The controller equation for control loop I was

$$Q_c - Q_{C_{ss}} = K_1 (C_{\text{set}} - C) + K_2 \int_{t_1}^{t_2} (C_{\text{set}} - C)\, d\tau + K_3 \frac{d}{dt} (C_{\text{set}} - C) \tag{4}$$

The controller equation for control loop II was

$$Q - Q_{ss} = P_1 (T_{\text{set}} - T) + P_2 \int_{t_1}^{t_2} (T_{\text{set}} - T) + P_3 \frac{d}{dt} (T_{\text{set}} - T) \tag{5}$$

The object of the research was to develop an adaptive control scheme for use when there were uncontrolled parameter drifts in the system. To simulate these parameter drifts, the coefficient k_o in Equations (2) and (3) was replaced by

$$k_o = k_{o_{\text{initial}}} (1.0 + ACT) \tag{6}$$

and the cooling water temperature was assumed to behave according to

$$T_c = T_{c_{\text{initial}}} + C_1 \sin w_1 t + C_2 t \tag{7}$$

Equation (6) was intended to simulate the phenomenon of catalyst activity drift, even though in the simple model used no catalyst was involved in the reaction. Equation (7) was intended to simulate a cycling of cooling water temperature plus a linear drift which might be thought of as a periodic cycling between day and night temperatures plus a seasonal drift. When the values of K_c and T_c were allowed to change according to Equations (6) and (7) in the absence of any adaptive control method, the overall system degraded, eventually reaching instability. Thus the need for adaptive control in the system was real.

The entire system, including the superimposed adaptive control systems discussed later in this paper, was simulated on the CDC 3400 Control Data Computer at the Northwestern University Computing Center. Equations (2), (3), (6), and (7) were used for the simulation only. Generally, they would not be known. Thus, as will be demonstrated, no information contained in them except the normally measurable values of the concentration and temperature was used to implement the adaptive control system.

MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

Figure 2 is a simplified functional diagram of the model reference adaptive control system. The theory of such a model reference system is presented by Osborn (*13*). A brief summary of the theory is presented here. The model reference adaptive control system operates by minimizing a performance index given by

$$PI = \frac{1}{2} \int_0^t e^2(\tau)\, d\tau \tag{8}$$

where e (called the response error) is the difference between the system response and the desired response to some input forcing function. The function $e^2/2$ was chosen for this study because its use resulted in an adaptive control system which was readily implemented. Theoretically any even function of the response error is suitable. At the minimum, that is, the best values of the adjustable parameters

$$\frac{\partial PI}{\partial K_i} = \int_0^t e \frac{\partial e}{\partial K_i}\, d\tau = 0 \quad (i = 1, 2, \cdots, n) \tag{9}$$

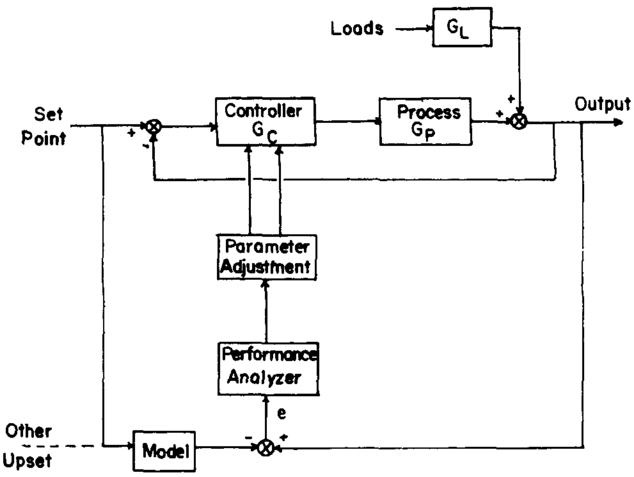

Fig. 2. Simplified diagram of the model reference adaptive control system.

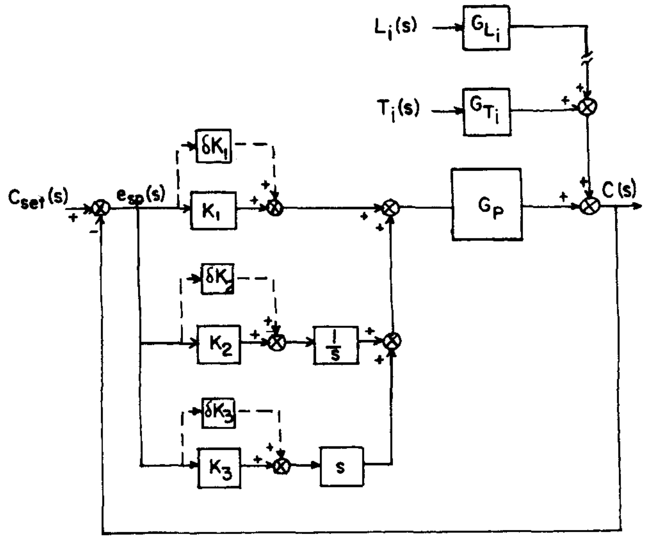

Fig. 3. Modified block diagram representation of the closed-loop control system.

The mathematical method of steepest descent suggests the means of modifying the adjustable parameters to achieve the minimum of a function; that is

$$K_i - K_{i0} = K_i = -F_i \frac{\partial PI}{\partial K_i} \quad (i = 1, 2, \cdots, n) \tag{10}$$

where F_i is a gain which determines the magnitude of the adjustment. From the definition of the response error and from noting that the desired response is independent of the adjustable parameters, Equation (9) can be rewritten as

$$\frac{\partial PI}{\partial K_i} = \int_0^t \frac{\partial C_{\text{system}}}{\partial K_i} (C_{\text{system}} - C_{\text{model}}) d\tau = 0 \tag{11}$$

Equation (11) states that the problem of determining the partial derivatives $(\partial PI/\partial K_i)$ needed to implement Equation (10) can be reduced to the problem of determining $(\partial C_{\text{system}}/\partial K_i)$.

The block diagram approach suggested by Whitaker (17) for determining approximations to the terms $(\partial C_{system}/\partial K_i)$ will be presented here. The first step is to arrange the conventional block diagram of a typical closed-loop control system so that the adjustable parameters appear as single blocks, as shown in Figure 3. The second step is to draw on the hypothesis that the effect of a change of δK_i in the adjustable parameter K_i is equivalent to the admission of a signal equal to $e_{sp_i} \delta K_i$ immediately after the block K_i. The signal e_{sp_i} is the signal which drives the block K_i. By conventional reduction of the block diagram shown in Figure 3

$$C_{\text{system}}(s) = \frac{G_p G_c}{1 + G_p G_c} C_{\text{set}}(s) + \frac{G_p}{1 + G_p G_c} \delta K_1 e_{sp}$$

$$+ \delta K_2 \frac{e_{sp}}{s} + \delta K_3 s e_{sp} + \frac{G_{Ti}}{1 + G_p G_c} Ti + \sum_j \frac{G_{Lj}}{1 + G_p G_c} Lj \tag{12}$$

where G_c is the standard representation for the PID controller. Taking the view that the resulting expression for C_{system} is made up of the effect of a disturbance in the input plus incremental disturbances in the K_i's, the increment in C_{system} due to δK_1, δK_2, and δK_3 is δC_{system}. In the limit

$$\frac{\partial C_{\text{system}}}{\partial K_1} = G_{\text{system}} \frac{1}{G_c} e_{sp}(s) \tag{13}$$

$$\frac{\partial C_{\text{system}}}{\partial K_2} = G_{\text{system}} \frac{1}{G_c} \frac{e_{sp}(s)}{s} \tag{14}$$

$$\frac{\partial C_{\text{system}}}{\partial K_3} = G_{\text{system}} \frac{1}{G_c} s e_{sp}(s) \tag{15}$$

where

$$G_{\text{system}} = \frac{G_p G_c}{1 + G_p G_c} \tag{16}$$

To accomplish this development in terms of Laplace transforms, the first assumption in the development of the model reference adaptive control system is necessary. It is required to assume that the time rate of change of all parameters in the system, including the adjustable parameters K_i's, equals zero. That is $dK_i/dt = 0$ and $dW_j/dt = 0$, $(1 = 1, 2, \cdots, n; j = 1, 2, \cdots, p)$.

Equations (13) through (15) are still not usable for implementing the adaptive control system because G_p is not generally known. The second assumption in the development of the model reference system is now made. It is assumed that some simple linear model is a useful approximation to G_{system} for the determination of approximations $(\partial C_{\text{system}}/\partial K_i)$. Incorporating the term $1/G_c$ into the model since it is a known transfer function, we can write Equations (13) through (15) as

$$\frac{\partial C_{\text{system}}}{\partial K_1} \approx G_{\text{model}}\ e_{sp}(s) \tag{17}$$

$$\frac{\partial C_{\text{system}}}{\partial K_2} \approx G_{\text{model}} \frac{e_{sp}(s)}{s} \tag{18}$$

$$\frac{\partial C_{\text{system}}}{\partial K_3} \approx G_{\text{model}}\ s e_{sp}(s) \tag{19}$$

The alternative to using G_{model}, identifying G_{system}, is neither necessary nor practical. If the overall system is close to the desired system, G_{model} is a good representation. If the overall system is significantly different from the desired system, the object is to change G_{system} as rapidly as possible to the desired value. Identification schemes will not track this change unless the change to G_{system} is so slow as to make the adaptive control method impractical.

Having assumed that $dK_i/dt = 0$, it would appear that to use this method of model reference adaptive control, the time rate of change in the adjustable parameters must be negligible during the transient response interval over which the partial derivatives $(\partial C_{\text{system}}/\partial K_i)$ are evaluated. In original investigations of model reference systems, it was shown to be advantageous to perform the parameter adjustment during this transient (*13*). Making appropriate substitutions in Equation (10), differentiating with respect to time, and assuming that the limits of integration are independent of time

$$\frac{dK_i}{dt} = -F_i \frac{\partial C_{\text{system}}}{\partial K_i} (C_{\text{system}} - C_{\text{model}})_{i=1,2,\cdots n} \quad (20)$$

One can use Equation (20) for modifying the adjustable parameters.

The assumptions made in the development of the theory of the model reference adaptive control system are good assumptions when the adjustable parameters are close to their proper values; $dK_i/dt \approx 0$ since no adjustment is required, and $G_{\text{system}} \approx G_{\text{model}}$ because the system characteristics resemble those of the model. The workability of the model reference adaptive control system when the values of the adjustable parameters are far from their optimum depends on whether or not the approximations used to generate $(\partial C_{\text{system}}/\partial K_i)$ result in changes in these parameters which are in the proper direction, thus permitting continued adjustments toward the desired values.

That the model reference adaptive control system discussed above was applicable to a typical nonlinear chemical processing situation is demonstrated in the remainder of this paper (*15*).

SINGLE-CONTROL LOOP APPLICATION

The first step in the implementation of the model reference adaptive control system was to define the optimum or desired dynamic performance. Preliminary simulation showed that the inlet concentration was the load variable which placed the largest demands on the conventional control system. A definition of the desired response was accomplished by a trial and error setting of the parameters in Equation (4), such that the response shown in Figure 4 for a large step upset in the critical load variable, the inlet concentration, resulted. In the operation of a large continuous process, this response would be quite acceptable. It was then assumed that the inlet concentration signal was not available for use as a test signal to actuate the adaptive control system. Thus the desired response to a step upset in the inlet concentration was related to a step upset in the inlet temperature, a variable which was assumed to be available for use as a test signal. Figure 4 also shows the initial dynamic response to a −5°F. step upset in the inlet temperature. This was considered to be the desired response for the adaptive control system. A few quick trials indicated that linear transfer function

$$C_{\text{model}}(s) = \frac{-2.06 \times 10^{-7}\, s}{(s + 0.029169)\left(s^2 + \frac{1.4}{60}\, s + \frac{1}{3600}\right)}\, T_L\,(s) \quad (21)$$

could be inverted to provide a near perfect approximation to the response of the actual system. Thus the adaptive control system's function was to minimize the performance index given by

$$PI_C = \frac{1}{2}\int_0^t (C_{\text{system}} - C_{\text{model}})^2 d\tau \quad (22)$$

for a step upset in the test signal variable, the inlet temperature, used to actuate the system. C_{model} is given by the inversion of Equation (21) and C_{system} is the actual system response described by Equations (2) through (7).

The second step in the implementation was to develop the approximations to $(\partial C_{\text{system}}/\partial K_i)(i = 1, 2, 3)$ needed in Equation (20). Note that the $G_{\text{model}}(s)$ presented in Equations (17) through (19) is not the same as that given in Equation (21). G_{model} in Equations (17) through (19) is

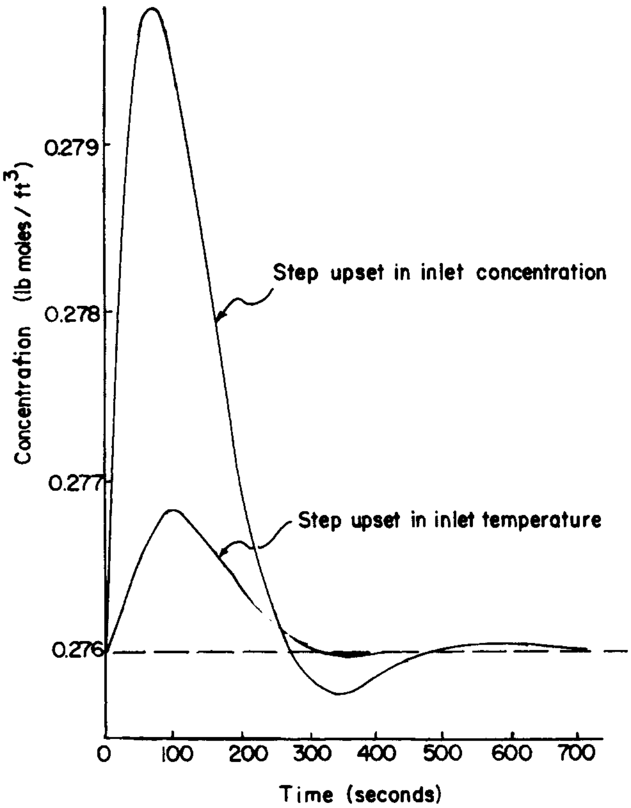

Fig. 4. Desired dynamic responses of outlet concentration to step inputs in load variables.

given by $(G_p/1 + G_p G_c)$, In the simulation an assumption was made that

$$\frac{G_p}{1 + G_p G_c} = -M \frac{G_{T_i}}{1 + G_p G_c} \quad (23)$$

where M is some positive constant. Thus Equations (21) and (23) were used in Equations (17) through (19) to establish the estimates $(\partial C_{\text{system}}/\partial K_i)$.

The third step in the implementation was to establish the frequency of operation of the adaptive control system. Intermittent operation of the adaptive control system was envisioned because, in the usual chemical process, the uncontrollable parameter drifts which occur are generally slow, taking place over a period of days or even weeks. Hence, adjustment of the controller parameters would be required only intermittently rather than continuously. Any number of criteria could be used to initiate the use of the adaptive control system. This problem was not considered in this work. Instead, the test signal upset in the inlet temperature shown in Figure 5 was initiated when the parameters k_o and T_c in the simulated process were changed to make adaptation necessary. The test signal which actuated both the process and the model was continued until adaptation was complete; that is, when the integral of the

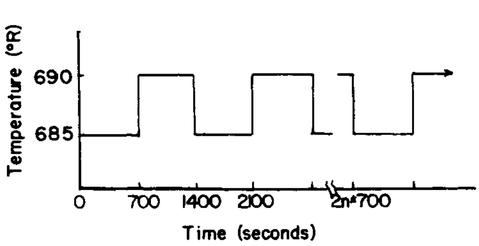

Fig. 5. Test signal upset in the inlet temperature used to actuate the adaptive control system.

absolute value of the response error over one complete transient was lower than some preset value.

The final step required to implement the adaptive control system was to establish the values of the adaptive loop gains required in Equation (20). (The constant $M \times 2.06 \times 10^{-7}$ was incorporated into these gains, thus making it unnecessary to determine the value of M.) Difficulty was encountered in determining gains which were applicable to the wide range of conditions simulated. The difficulty was attributed primarily to the problems associated with multiplying a large response error e by a large value of $(\partial C_{\text{system}}/\partial K_i)$. It was believed that this difficulty would impair the applicability of this technique. Therefore it was decided to investigate the use of a normalized version of Equation (20), using the sign of $(\partial C_{\text{system}}/\partial K_i)$ rather than its actual value. This eliminated the multiplication of two large values. Using the normalized version, the product $e \times (\partial C_{\text{system}}/\partial K_i)$ still becomes small as the system characteristics approach the model characteristics, since the value of e becomes small.

In the normalized version, initial estimates of proper gain values were made based on a priori estimates of the maximum response error expected, the time of the transient resulting from the test signal upset, and the amount of correction expected to be required in the adjustable parameters. These initial estimates of workable gain values were tried and found to work extremely well for a case in which the initial system response was moderately different from the desired response. Further tests showed that a wide range of gain values provided good adaptive loop performance if, initially, the system characteristics were such that the system's unadapted dynamic response was similar to the unadapted response used to test the initial gain estimates. Thus it was not surprising that the first estimates worked well. Upper limits to the gain values were found, above which the overall system was unstable. Also, gain values which were too low resulted in excessive time requirements for adaptation. Because the range of workable gains was wide, the problems concerned with instabilities or slow adaptation were easily avoided.

Further preliminary simulations showed that if the system characteristics were such that, initially, the system response was far removed from the desired response, different values of adaptive control loop gains were required for good adaptive control loop performance. These results necessitated the use of some means of adjusting the gains based on some measure of the response error. A method was devised which used the integral of the absolute value of the response error over one transient to establish the proper gain values.

The results presented in Figures 6 and 7 demonstrate the success of the model reference adaptive control system with suitable gain adjustment procesures in simultaneously adjusting the three controller parameters in the PID controller used to maintain the outlet concentration in the simulated chemical reactor system. In each figure the original model response provides a reference to the amount of adaptation required. The numbers n indicate that the response was experienced during the n^{th} test signal upset. Figure 6 shows that a moderate amount of adaptation was achieved in only 2,100 sec. A considerably more serious upset is presented in Figure 7. Note that during the first test signal upset, good stabilization of the process resulted. Excellent adaptation was achieved in 3,500 sec. despite the nearly unstable original condition of the process. These times are considered feasible for an adaptive control system operating in a real application.

APPLICATION TO INTERACTING CONTROL LOOPS

The interacting control loop situation was simulated by assuming control Loops I and II were in operation simultaneously (Figure 1). One would desire that an adaptive

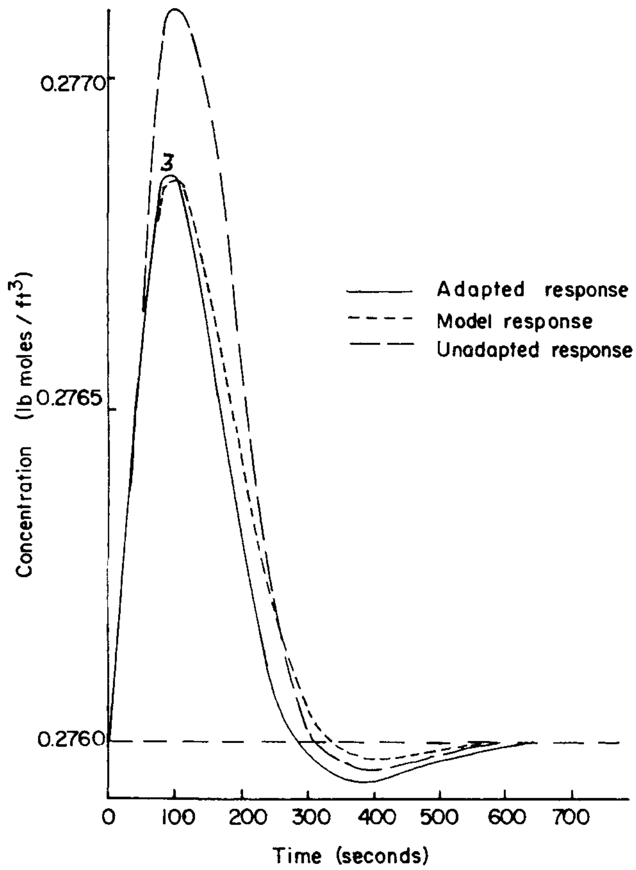

Fig. 6. Dynamic response of the outlet concentration demonstrating successful adaptation of controller parameters. Moderate amount of adaptation required.

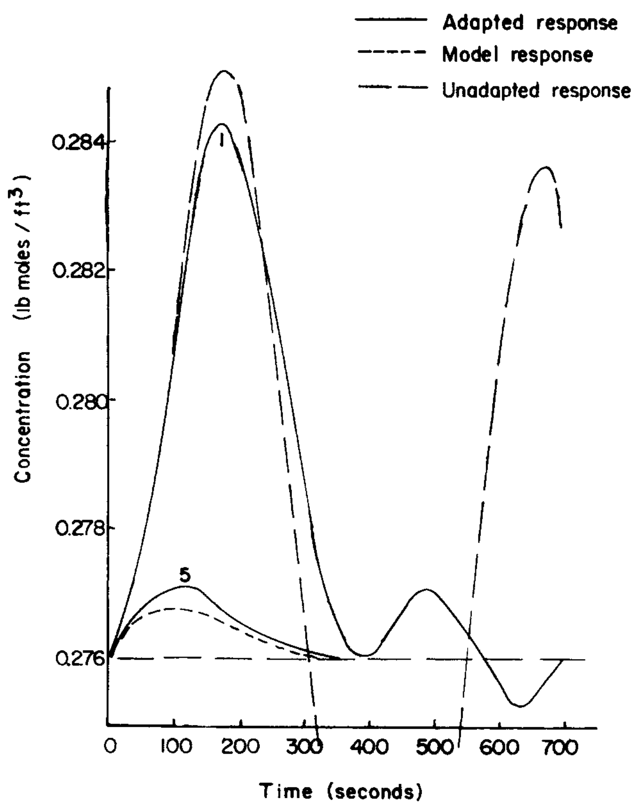

Fig. 7. Dynamic response of the outlet concentration demonstrating successful adaptation of controller parameters. System initially unstable.

control loop superimposed on each conventional control loop would operate satisfactorily despite the interactions between the control loops. Thus equations similar to those developed for the concentration control loop were developed for the adaptive loop which was superimposed on the conventional temperature controller.

The first step was again to develop the models, that is, the responses considered desirable. Preliminary simulations showed that in the interacting case, the critical load variable was the inlet temperature. Thus the model responses could be developed directly from the acceptable responses to an upset in the inlet temperature. By trial and error, the six parameters in the conventional controllers were set such that the desired response of the concentration to an upset in the inlet temperature was approximated by inversion of Equation (21). A good approximation to the desired temperature response to the same upset was approximated by the inversion of the equation

$$T_{\text{model}}(s) = \frac{2.10 \times 10^{-4} s}{(s + 0.03571)\left(s^2 + \frac{1.2}{42} s + \frac{1}{.1764}\right)} T_i(s) \quad (24)$$

The desired model temperature temperature response is shown as part of Figure 8. The temperature adaptive control loop was to minimize the performance index given by

$$PI_T = \frac{1}{2}\int_0^t (T_{\text{system}} - T_{\text{model}})^2 d\tau \quad (25)$$

The equations which were required to implement the adaptive control loop for the temperature controller were developed in exactly the same manner as those used for the concentration controller. The same assumption stated in Equation (22) was again made. The equations used for the adaptive loop for the concentration controller were identical to those used in the single-loop case.

The same intermittent operation of the adaptive system was envisioned and the same test signal upset shown in Figure 5 was used to actuate the adaptive systems.

No attempt was made to use the non-normalized version of Equation (20). The normalized equations were used and initial estimates of the proper gains to be used in the adaptive temperature loop were made in a similar manner to that used in the estimation of the gains for the single-loop case. A similar method of adjusting the gains in the temperature loop based on the integral of the absolute value of the temperature response error over one transient was used. The same gain values and adjustment procedures developed for the concentration control system in the single-loop case were used in the interacting case.

Before presenting the results of this study, a clear understanding of the interactions between the loops is desirable. Adjustments to the concentration controller parameters affected the flow rate of the cooling water, which was itself a load variable in the temperature control loop. Thus the response of the temperature was affected by the adaptive control loop operating on the concentration controller. It should also be noted that the cooling water flow disturbance did not pass through the model as the test signal upset did. Hence adaptation in the temperature loop had to be accomplished despite an extraneous load upset which was affecting the system dynamic response. Simultaneously, the same effects were occurring in the concentration control loop due to the adaptation taking place in the temperature control loop.

Figures 8 and 9 are representative of the results obtained for the interacting control loops. Note that good adaptation was achieved in less than 1 hr. of adaptive loop operation. Figure 9 shows that the adaptation in the temperature loop was not as good as would be desired, however. Examination of Equations (2) through (7) indicated that only five of the six parameters in the conventional controllers were

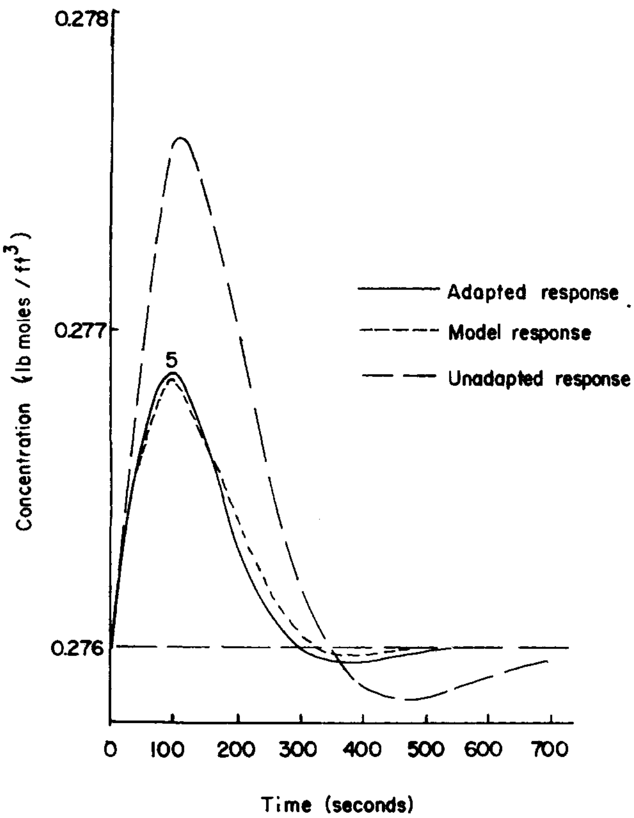

Fig. 8. Dynamic response of the outlet concentration demonstrating successful adaptation in the presence of interactions. Simultaneous adjustment of six controller parameters.

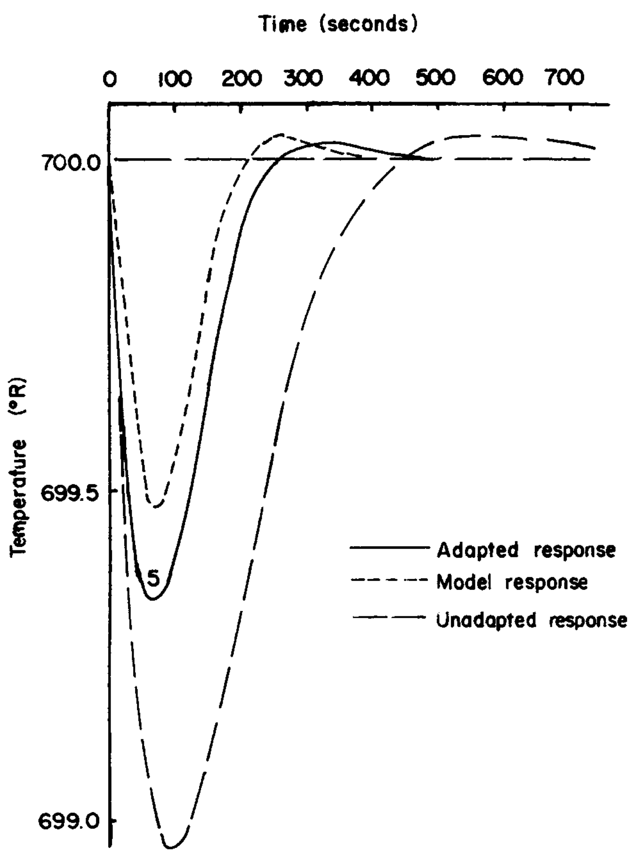

Fig. 9. Dynamic response of the reactor temperature demonstrating fair adaptation in the presence of interactions. Simultaneous adjustment of six controller parameters.

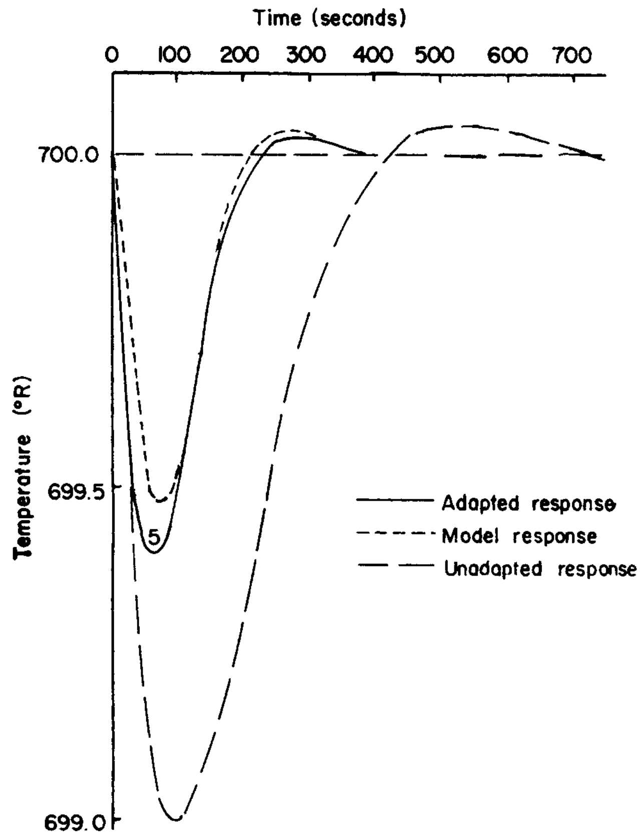

Fig. 10. Dynamic response of the reactor temperature demonstrating improved adaptation in the presence of interactions. Simultaneous adjustment of five controller parameters.

unique. When Equations (2), (3), and (4) were used to replace the term $d(T_{set} - T)/dt$ in Equation (5) with its equivalent, only the ratios P_1/P_3 and P_2/P_3 appeared. All runs were then repeated holding P_3 constant at its initial value and using the adaptive control loops to adjust the remaining five parameters. Figure 10 shows results comparable to those shown in Figure 9. Note the improved temperature adaptation. Adaptation of the concentration controller parameters was equally as good as in the first attempts at simultaneous adaptive control of the two loops.

STABILITY OF THE ADAPTIVE CONTROL SYSTEMS

Braun and Truxal (*14*) state that the results of work done to develop conditions for predicting stability of model reference adaptive control systems have not been encouraging and conclude that this is not a very promising area of endeavor. However, this does not mean that the question does not need investigation. The results discussed above showed that instabilities could result if adaptive loop gains were too high, but that the range of acceptable gains was wide enough to permit application of the method. Use of a gain adjustment procedure based on the degree of adaptation required further increased the set of system characteristics that could be adapted to approximate the desired system characteristics without stability problems.

That good adaptive control loop gains existed and were easily found was not the only question which would be of concern in the application on this technique to a real chemical processing situation. All the above work was done with all load variables except the test signal variable (and the control variables in the interacting case) held constant at their steady state values during operation of the adaptive control systems. This would be the situation which would exist in a process under good control. However, there is never any guarantee that another load upset would not occur during the time that the adaptive loops were in operation. It is critical that in this situation the adaptive loops would not cause the stable overall system to become unstable.

To investigate this, several runs were simulated where in each run, a large, extraneous step upset in the inlet concentration was initiated at some time during the period of operation of the adaptive control systems. In each case investigated, both in the single-loop situation and in the interacting situation, the overall system was highly stable despite the large extraneous upset which affected the system response but not the model response. Adaptation was continued after the extraneous upset, and adaptation was generally accomplished in five test signal step upsets.

CONCLUSIONS

With a simulated chemical process, a normalized version of model reference adaptive control, including suitable procedures for modifying adaptive loop gains, was used successfully to adjust conventional controller parameters to maintain good dynamic process response in the face of uncontrolled process parameter drifts. In the cases studied adaptation was sufficiently rapid to make the method practical for use in a real application. Stability of the overall control system was excellent.

Since the assumptions made in the development of the theory of the model reference adaptive control method were sometimes violated in the actual cases studied, the general applicability of the method cannot be guaranteed. However, it was shown to be highly effective over a wide range of required controller adjustments is cases very similar to those arising in typical large continuous processes. Thus it seems that the method merits consideration for implementation whenever adaptive control is desired.

It should be noted that the model reference method has the disadvantage that the process must be slightly upset to activate the adaptive control action and to achieve the controller parameter modification required. This disadvantage might be overcome by incorporating adaptive control with optimizing supervisory control and making use of the set point adjustments which result from optimization calculations to activate the adaptive system.

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NOTATION

A = cooling coil area, sq. ft.
A = reactant
ACT = term which simulates catalyst activity drift
B = product
C = concentration, lb.-mole/cu. ft.
C = constants in equation simulating cooling water temperature drift
C_p = specific heat, B.t.u./(lb.)(°R.)
e = response error
e = signal preceding adjustable parameter in control loop
F = gain in adaptive control loop
G = transfer function
ΔH = heat of reaction, B.t.u./lb.-mole
K = adjustable controller parameter in concentration control loop
k = Arrhenius rate constant, sec.$^{-1}$
L = Laplace transform of load variable upset function
M = positive constant
P = adjustable controller parameter in temperature control loop

PI = performance index
Q = reactant flow rate, cu. ft./sec.
Q_c = cooling water flow rate, cu. ft./sec.
R = gas constant, B.t.u./(lb.-mole)(°R.)
s = Laplace transform variable
T = temperature, °F. or °R.
t = time
U = overall heat transfer coefficient, B.t.u./(sec.)(sq. ft.)(°R.)
V = volume, cu. ft.
W = process parameters excluding adjustable controller parameters

Greek Letters

ρ = density, lb./cu. ft.
τ = time
w = frequency

Subscripts

C = concentration
C = controller
c = cooling water
i = inlet
i = index
j = index
o = initial value
p = process
Sp = signal preceding adjustable parameter in control loop
ss = steady state
set = set point
T = temperature

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Parameter Sensitivity of Systems Described by Nonlinear Ordinary Differential Equations

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An approximate analytical method is developed to estimate the parameter sensitivity of the solution of a set of nonlinear ordinary differential equations describing a system which exhibits periodic behavior. An approximate solution is constructed in terms of both the approximate periodic solution determined from Galerkin equations and the envelope and phase of the oscillation away from such a periodic solution. Parameter sensitivity information is then obtained by examining the parameter variation effect on the approximate solution.

Examples of two- and three-dimensional nonlinear systems illustrate this procedure and show that the effect of parameter change on the solution is predicted with sufficient accuracy to make this method useful for nonlinear analysis.

The behavior of an engineering system under normal operating conditions is influenced by parameter disturbances. If an analytical solution to the set of nonlinear ordinary differential equations describing such a system were available, the effect of parameter changes could be determined in a straightforward manner. However, it is usually difficult or impossible to obtain an analytical solution to nonlinear differential equations. This difficulty has motivated the use of approximations for the analysis of a nonlinear system. Linearization of the set of differential equations is valuable, but it frequently fails to describe the system behavior in a region not very far from the singular point about which the linearization is performed. Fur-

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